Egyptian Journal of Chemistry
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Soret and Dufour Effects with Hall Currents on Peristaltic Flow of Casson Fluid with Heat and Mass Transfer Through Non-Darcy Porous Medium Inside Vertical Channel

W. Abbas[a,] *, Nabil T. M. Eldabe[b], Rasha A. Abdelkhalek[c], Nehad A. Zidan[c], S.Y.Marzouk[a]

[a] *Basic and Applied Science Department, College of Engineering and Technology, Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt*
[b] *Department of Mathematics Faculty of Education Ain Shams University. Cairo, Roxy, Egypt*
[c] *Physics and Engineering Mathematics Department, Faculty of Engineering-Mattaria, Helwan University, Cairo, Egypt*

Abstract

The peristaltic flow with heat and mass transfer for non-Newtonian fluid through non- darcy porous medium is investigated. The fluid obeys Casson model and the effects of Hall currents, Ohmic and viscous dissipations, heat generation and chemical reaction are taken into account. Problem is mathematically modulated using a system of partial differential equations describing velocity, temperature and concentration of the fluid. The non-dimensional partial differential equations are simplified using the approximations of long wavelength and low Reynolds number. Then this system subjected to appropriate boundary conditions is solved by using homotopy perturbation method. The effect of obtained solutions of velocity, temperatures and concentration as functions of the physical parameters of the problem are discussed computationally and illustrated graphically. It is shown that the velocity decreased by increasing of the magnetic field, non –Newtonian parameter, while it increases with non- darcian parameter and heat generation, also, the temperature decreases with increasing of Eckert and Grashoff numbers while it increases with heat generations. Dufour number and non-direction furthermore, the concentration decreases with Newtonian and magnetic parameters, while it increases with chemical reaction and Eckert number.

Key of words: Casson fluid, Heat and Mass transfer, non-darcy porous medium, Peristaltic transport.

1. Introduction

Peristaltic transport is a form of fluid transport via travelling waves imposed on the walls of a distensible fluid such as transporting urine from kidney to bladder, movement of food through esophagus, the vasomotion of small blood vessels, and chyme motion in the intestine [1]. Also, peristaltic transport has vital in several applications in industrial such as transporting corrosive fluids, roller pumps, and sanitary fluids [2]. Earlier several researchers have carried out the studied peristaltic flow with heat and mass transfer under different physical conditions. Nadeem et al. [3] investigated the peristaltic flow of non-Newtonian third order fluid with heat and mass transfer analysis for a diverging tube. The effect of heat and mass transfer on the two dimensional peristaltic flow of a Johnson Segalman fluid considered induced magnetic field was studied by Nadeem and Akber [4]. Hayat and Hania [5] reported the peristaltic flow with heat mass transfer for Williamson fluid for non-uniform channel under slip conditions. Vaidya et al. [6] analysed peristaltic motion with effects of heat transfer for non-Newtonian Herschel-Bulkley fluid through a porous elastic tube. A peristaltic blood flow of non-Newtonian Carreau fluid with heat transfer phenomenon through a curved channel was discussed by Tanveer et al. [7].

On the last few years several researchers studied fluid flow through Magneto hydrodynamic (MHD) in bio-medical and industry such as usage the magnetic field as a blood pump in carrying out cardiac

*Corresponding author e-mail: wael_abass@aast.edu.
Receive Date: 09 April 2021, Revise Date: 16 April 2021, Accept Date: 27 April 2021
DOI: 10.21608/EJCHEM.2021.71400.3576

operations, and giant Magneto resistance sensors [8]. Mekheimer [9] analysed peristaltic transport for MHD flow has couple stress fluid flow with an effect of an induced magnetic field. The two-dimensional peristaltic transport with heat and mass transfer of Non-Newtonian micro polar fluid under wall properties effect was studied by Eldabe, and Abou-Zaid [10]. Abbasi et al. [11] reported the MHD peristaltic transport of Carreau–Yasuda fluid taken Hall effects in a curved conduit. Influence of Hall current, homogeneous–heterogeneous reaction and Joule heating for non-Newtonian third order fluid on peristaltic flow in a channel was studied by Hayat et al. [12]. Bhatti et al. [13] presented MHD heat and mass transfer on the peristaltic motion for Sisko fluid flow in the presence of chemical reaction. Nawaz et al. [14] discussed the entropy generation of peristaltic motion with Soret and Dufour and slip conditions effect for Williamson fluid with radial magnetic field in curved channel.

The main aim for the present work is investigating the combined effects of Soret, Dufour, Ohmic and viscous dissipations on the peristaltic flow of Casson fluid with heat and mass transfer. The system of resulting equations that govern the model is solved semi-analytically using homotopy perturbation technique. A set of graphs were used in analyzing results and conclusion. The paper has been presented sections wise as: The mathematical models and formulation of the problem is revealed in Section 2. Section 3 explained the solution methodology. Section 4 Discusses results. Section 5 depicted the Concluding remarks.

2. Mathematical Model

2.1. Description of the Problem

The peristaltic flow of an incompressible, electrically conducting non-Newtonian Casson's fluid past a porous medium for a two-dimensional symmetric flexible channel of width $d_1 + d_2$ is investigated. A physical model is described in Fig. 1. Taking that, X-axis as the direction flow and considering Y-axis as normal to the flow. $a_1$and b_1 are amplitude of sinusoidal wave propagates along the channel walls with uniform speed $\boldsymbol{V_c}$ at direction of X-axis. Appling strong constant magnetic field with flux density$\vec{B} = (0, 0, B_o)$, taking Hall effects. Neglecting the induced magnetic field by assuming a very small magnetic Reynolds number. Assuming T_o and $\mathbf{C_o}$ are temperature and concentration of fluid at the right wall while T_1 and C_1 are temperature and concentration of fluid at the left wall. Representing the wall geometry as:

$$\boldsymbol{Y(x,t) = \pm H = \pm(d + a\cos[\tfrac{2\pi}{\lambda}(x - V_c t)])} \quad (1)$$

Where : a ,b, $\boldsymbol{\lambda}$ are waves amplitudes and wave length

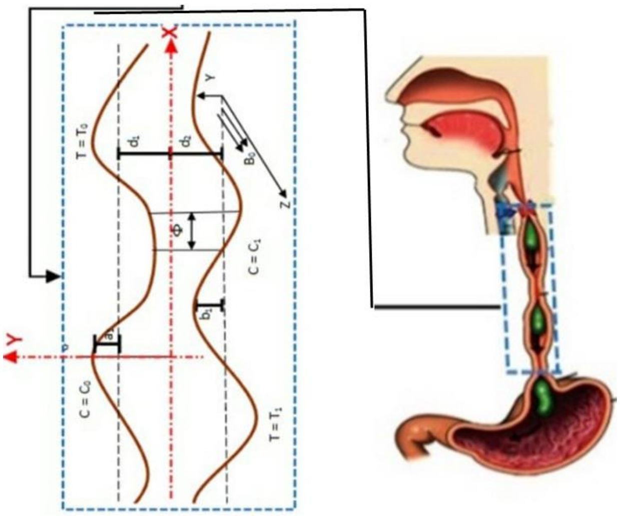

Fig.1: Geometry of the problem

2.2. Governing Equations

The basic equations that govern the continuity, momentum, heat and concentration of the fluid can be written as [5]:

Continuity equation:

$$\nabla . V = 0 \quad (2)$$

Momentum Equation:

$$\rho \frac{\partial \vec{V}}{\partial t} = -\nabla \vec{P} + \nabla . \vec{\tau} + \mu_e (\vec{J} \times \vec{B}) + \vec{F_P} + \vec{F_C} + \vec{F_T} \quad (3)$$

eat equation:

$$\rho C_P \frac{\partial T}{\partial t} = k\nabla^2 T + \frac{D_m K_T}{C_S} \nabla^2 C + \frac{1}{\sigma} J.J + \Phi + Q(T - T_0) \quad (4)$$

Concentration equation:

$$\frac{\partial C}{\partial t} = D\nabla^2 C + \frac{D_m K_T}{T} \nabla^2 T - K_C(C - C_0) \quad (5)$$

Where, ρ is the fluid density, $\vec{V}$ is the velocity vector of fluid, P is the pressure, F_p is the porous force, F_C is the thermal expansion due to concentration, , F_T is thermal expansion due to temperature, $C_P$is the specific heat for constant pressure, T is the temperature, k is thermal conductivity, $D_m$is the coefficient of mass diffusivity, $k_T$is the thermal diffusion ratio, $C_s$is concentration susceptibility, C is the concentration, σ is the electric conductivity, Φ is viscous dissipation, Q is heat source constant,

$\vec{J}$ is current density which is formulation using the generalized Ohm's law [15-17]:

$$\vec{J} = \sigma[\vec{E} + \vec{V} \times \vec{B} - \gamma(\vec{J} \times \vec{B})] \quad (6)$$

Where, γ is the Hall factor. Assuming no applied or polarization voltage so the total electric field ($\vec{E} = 0$). Lorentz force can be representing as:

$$\vec{J} \times \vec{B} = \frac{-\sigma B_0^2}{(1+m^2)}[(U - mV)\vec{\iota} + (mU + V)\vec{J} \quad (7)$$

Where, U and V are the X and Y components of the velocity vector, $m = \sigma \gamma B_o$ is Hall parameter.

The equation of isotropic rheological for incompressible Casson fluid flow [18, 19] can be defined as:

$$t_{ij} = \begin{cases} 2\left(\mu_B + \frac{P_y}{2\pi}\right)e_{ij} & \pi > \pi_c \\ 2\left(\mu_B + \frac{P_y}{2\pi_c}\right)e_{ij} & \pi > \pi_c \end{cases}$$

Where, t_{ij} is shear stress, π is the product of the component of deformation rate with itself, π_c is the critical value, μ_B is plastic dynamic viscosity, e_{ij} is the deformation rate, and P_y is the yield stress of the fluid where:

$$P_y = \frac{\mu_B\sqrt{2\pi}}{\beta}$$

Where, β is the Casson's parameter.

Using the above assumptions governing equations are:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$\rho\left(\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = \frac{-\partial P}{\partial X} + \mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\sigma B_0^2}{1+m^2}(U - mV) + \rho g B_T(T - T_\infty) + \rho g B_C(C - C_\infty) - \frac{\mu}{k}U - \frac{\rho n}{k}\left(U\sqrt{(U^2+V^2)}\right) \quad (9)$$

$$\rho\left(\frac{\partial V}{\partial T} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = \frac{-\partial P}{\partial Y} + \mu\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\sigma B_0^2}{1+m^2}(Um - V) - \frac{\mu}{k}V - \frac{\rho n}{k}\left(V\sqrt{(U^2+V^2)}\right) \quad (10)$$

$$\rho C_P\left(\frac{\partial T}{\partial T} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = k\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \mu\left(1+\frac{1}{\beta}\right)\left(2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial X}\right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right) + \frac{D_m K_T}{C_S}\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right)\frac{\sigma B_0^2}{1+m^2}(U^2+V^2)Q(T - T_0) \quad (11)$$

$$\left(\frac{\partial C}{\partial T} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right) = D_m\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) + \frac{D_m K_T}{T_m}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right)K_C(C - C_0) \quad (12)$$

Using the transformations

$$x = X - V_c t,\ y = Y,\ u = U - V_c,\ v = V,\ p(x) = P(X,t) \quad (13)$$

Where, u and v are velocity components.

Consider the following non – dimensional quantities

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{d},\ \bar{u} = \frac{u}{v_c},\ \bar{v} = \frac{v}{\delta v_c},\ h = \frac{H}{d},\ \delta = \frac{d}{\lambda},\ \bar{P} = \frac{d^2 P}{\mu v_c \lambda},\ R_e = \frac{v_c d}{\gamma},\ \varphi = \frac{C - C_0}{C_1 - C_0},\ \theta = \frac{T - T_0}{T_1 - T_0},\ \epsilon = \frac{a}{d},\ \bar{\psi} = \frac{\psi}{v_c d},\ M = \frac{\sigma d^2 B_0^2}{\mu(1+m^2)},\ G = \frac{Q d^2}{\mu C_P},$$

$$E_c = \frac{v_c^2}{C_P(T_1 - T_0)},\ D_f = \frac{\rho D_m K_T(C_1 - C_0)}{\mu C_P C_S},\ S_r = \frac{D_m K_T(T_1 - T_0)}{\nu T_m(C_1 - C_0)},\ S_c = \frac{\nu}{D_m},\ \lambda = \frac{K_C d^2}{\nu},\ F = \frac{n V_C d^2}{\nu K}$$

Where ,R_eReynolds number, M is magnetic parameter isP_r is Prandtl number, S_c is Schmidt number, $S_r$is Soret number, E_c is Eckert number, D_F is Dufour number and F is non darcian parameter.

Introduce the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \ \ and\ v = -\frac{\partial \psi}{\partial x}$$

Using the non-dimensional variables the equations (8)-(12) can be written as:

$$\left[1 + \frac{1}{\beta}\right]\frac{\partial^3 \psi}{\partial y^3} - \left(M + \frac{1}{k}\right)\left(\frac{\partial \psi}{\partial y} + 1\right) - F\left(\frac{\partial \psi}{\partial y} + 1\right)^2 + G_r\theta + G_c\varphi = A \quad (14)$$

$$\left(\frac{\partial^2 \theta}{\partial y^2}\right) + P_r D_f\left(\frac{\partial^2 \varphi}{\partial y^2}\right) + M E_c P_r\left(\frac{\partial \psi}{\partial y} + 1\right)^2 + \left(1 + \frac{1}{\beta}\right)E_c P_r\left(\frac{\partial^2 \psi}{\partial y^2}\right) + G P_r\theta = 0^2 \quad (15)$$

$$\left(\frac{\partial^2 \varphi}{\partial y^2}\right) + S_c S_r\left(\frac{\partial^2 \theta}{\partial y^2}\right) - \lambda_0\,\varphi = 0 \quad (16)$$

Boundary conditions can be written as:

$$\text{at } y = h,\ \frac{\partial \psi}{\partial y} = -1,\ \theta = 1,\ \varphi = 1$$

$$\text{at } y = -h,\ \frac{\partial \psi}{\partial y} = -1,\ \theta = 0,\ \varphi = 0 \quad (17)$$

and $\psi = 0$ at $y = 0$

where $h = 1 + \epsilon\cos(2\pi x)$

3. Semi-analytical Solution

Many physics problems are modelled by differential equations which are difficult to have their exact solutions. The homotopy perturbation method is a way to have approximate solutions for nonlinear problems analytically. It was developed first by He [20, 21] and represented by Wu and He [22].

Solving equations (14-16) subjected to boundary conditions (17). The homotopy perturbation technique is applied, and the solutions obtained are functions of the physical parameters of the problem.

These solutions can be written as:

$$\psi = S_{50}\,y^{11} + S_{51}\,y^9 - S_{52}\,y^8 + S_{65}\,y^7 + S_{54}\,y^6 + S_{66}\,y^5 + S_{67}\,y^4 + S_{68}\,y^3 + S_{69}\,y^2 + S_{70}\,y \quad (18)$$

$$\theta = -S_{94}\,y^{10} + S_{95}\,y^8 + S_{96}\,y^7 + S_{102}\,y^6 + S_{98}\,y^5 + S_{103}y^4 + S_{104}\,y^3 - S_{105}y^2 + S_{106}y + S_{107} \quad (19)$$

$$\varphi = S_{114}\,y^6 + S_{115}\,y^5 + S_{116}\,y^4 + S_{121}\,y^3 + S_{122}\,y^2 + S_{123}y + S_{124} \quad (20)$$

Where $(S_0 - S_{124})$ are the functions of x and defined in the appendix.

4. Results and physical discussion:

In this section, the results for the peristaltic flow of Casson fluid in a symmetric channel considering heat and mass transfer are presented graphically and discussed for different various parameters on velocity, temperature and concentration distributions. The graphical results are illustrated in Figures (2-34).

Figures (2-12) illustrated the relations between the velocity field $u = \psi_y$ and the different parameters of the problem. It is clear that the velocity decreases by the increasing of magnetic field parameter M, the non –Newtonian Casson parameter $\beta$and the chemical reaction parameter λ_0 .These results well agreement physically with the effect of Lorentz force which retard the fluid motion, as well as the increasing of viscosity of non-Newtonian fluid decreasing velocity of fluid. Furthermore, the velocity increases with non-darcian parameter F, this due to the increased of the porosity of the medium. Also , the velocity increases with increasing the Grashoff G_r ,Prandtl P_r ,Eckert E_c numbers and heat generation G.This due to the fact that , the cohesion and coherence between the molecules of the fluid decrease with increasing the fluid temperature , which increases the flow of the fluid.

Figures (13 - 25)discussed the relations between the temperature θ and different values of the physical parameters of the problem , It shown that, temperature increases with the non-darcian parameter F, heat generation G , the Dufour parameter D_f while it decreases with λ_0 and Prandtl number P_r .Increasing of Casson parameter β , Eckert number E_c , Grashoff number G_r , modified Grashoff number G_c and magntic parameter M . Also , the relations between the concentration φ and the physical parameters of the fluid are illustrated through the figures (26- 34) .It is seen that the concentration decreases by increasing Casson parameter β ,magnetic field parameter M , Soret and Shimited numbers , and heat generations while it increases with increasing the chemical reactions λ_0 .

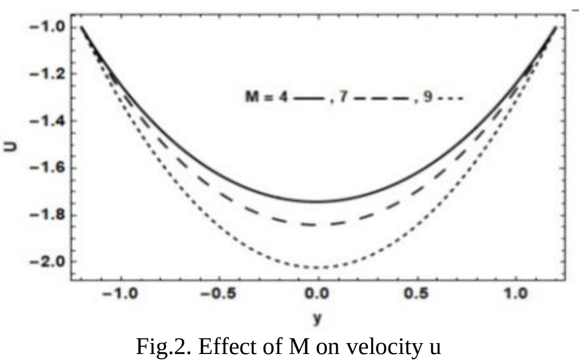

Fig.2. Effect of M on velocity u

Where k=0.6, F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2

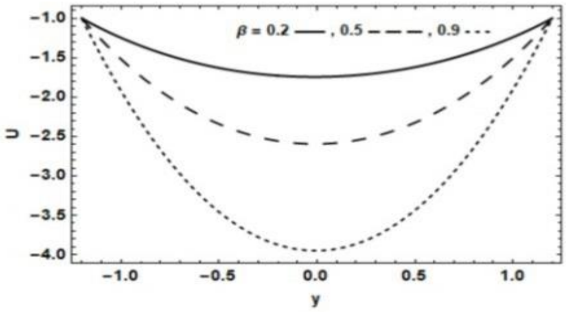

Fig.3. Effect of β on velocity u

Where k=0.6,F=0.2, G_r =0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3,M=4

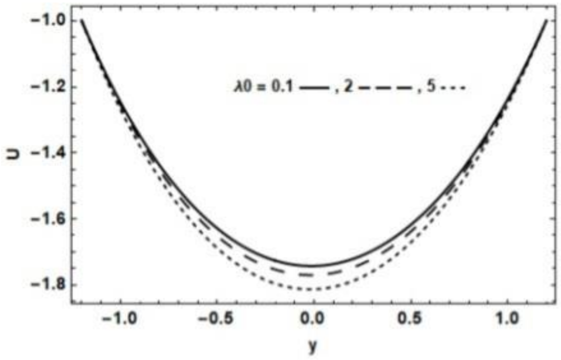

Fig.4. Effect of λ_0 on velocity u

Where k= 0.6,F= 0.2, G_r= 0.5, G_c=0.4, P_r= 0.6,G= 0.3, E_c= 0.6, D_f= 3, S_c= 3, S_r=3, β = 0.2, M=4

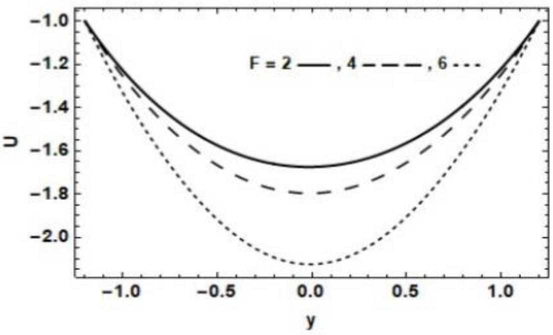

Fig.5Effect of F on velocity u

Where k= 0.6,G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r=3 , β = 0.2, M=4

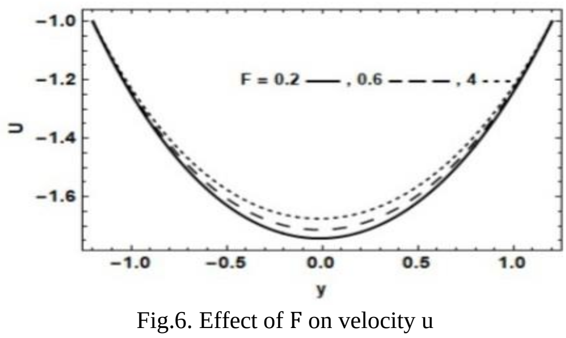

Fig.6. Effect of F on velocity u

Where k=0.6, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, , β = 0.2, M = 4

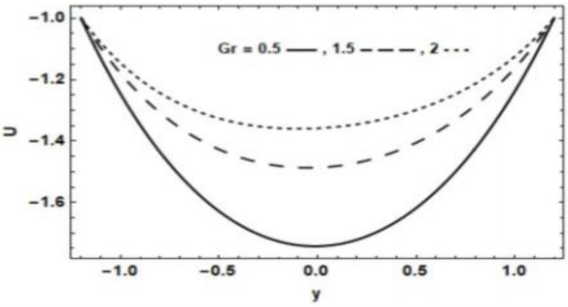

Fig.7. Effect of G_r on velocity u
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c=3, S_r=3 , $\beta = 0.2, M = 4$

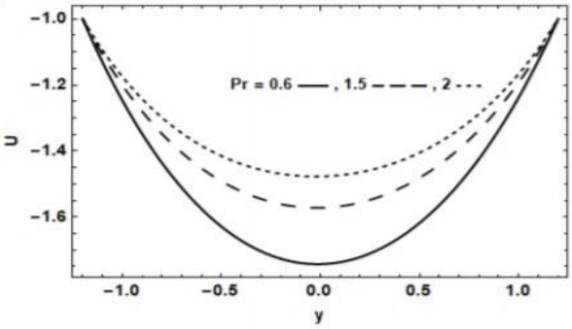

Fig.8. Effect of P_r on velocity u
Where k= 0.6, F= 0.2, G_r= 0.5, G_c= 0.4,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f=3, S_c=3, S_r=3 , $\beta = 0.2, M = 4$

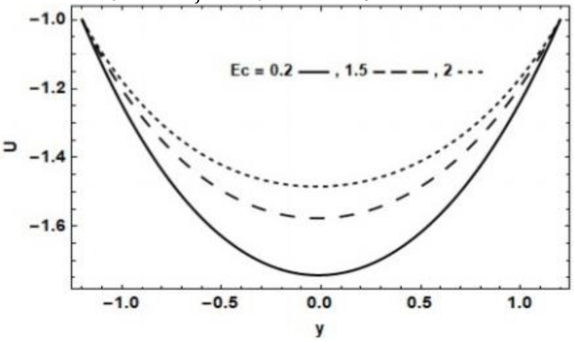

Fig.9. Effect of E_c on velocity u
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, λ_0= 0.1, D_f= 3, S_c=3, S_r=3, β = 0.2,M=4

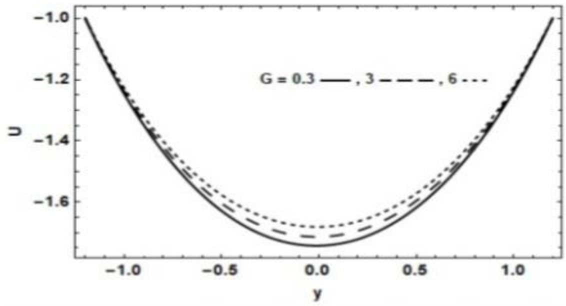

Fig.10. Effect of G on velocity u
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c=3, S_r=3, β = 0.2, M=4

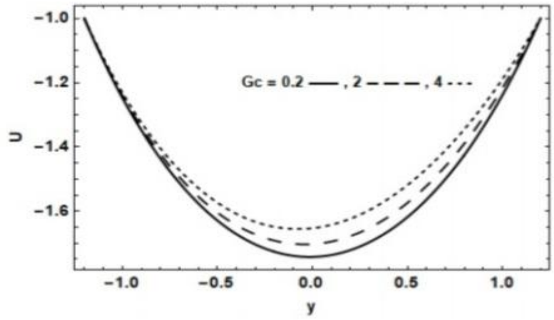

Fig.11. Effect of G_c on velocity u
Where k= 0.6,F= 0.2, G_r= 0.5, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f=3, S_c=3, S_r=3, β = 0.2, M=4

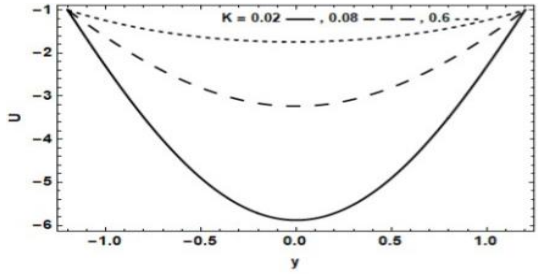

Fig.12. Effect of k on velocity u
Where F=0.2, G_r =0.5, G_c=0.4, P_r=0.6,G=0.3 , E_c=0.6, λ_0=0.1, D_f=3, S_c=3, S_r=3, β = 0.2, M=4

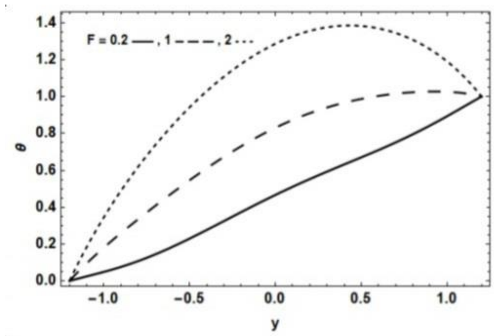

Fig.13. Effect of F on temperature θ
Where k= 0.6, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M=4

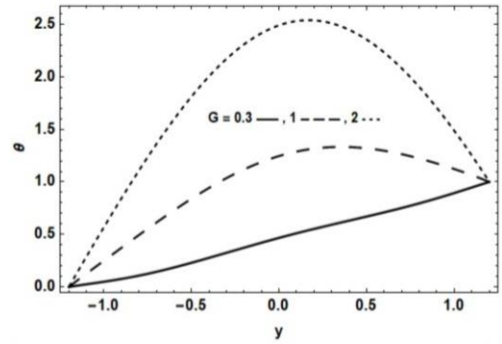

Fig.14. Effect of G on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r=3, β = 0.2, M=4

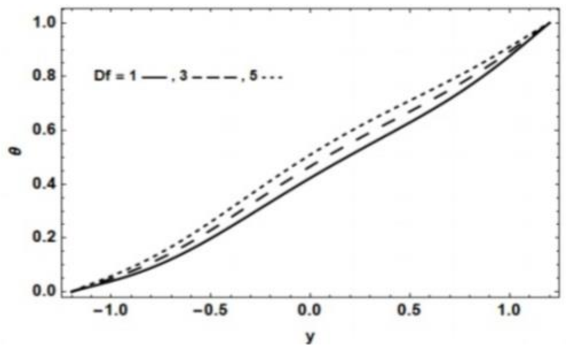

Fig.15 Effect of D_f on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, S_c= 3, S_r= 3, β = 0.2, M= 4

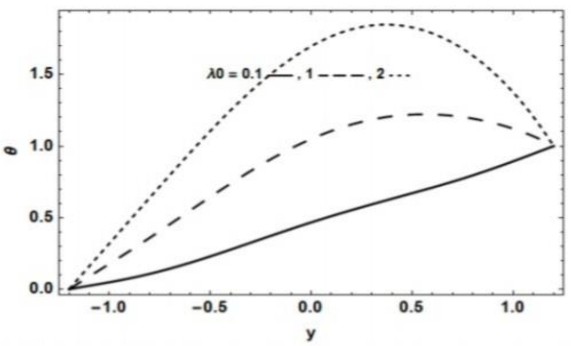

Fig.16. Effect of λ_0 on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, D_f=3, S_c= 3, S_r=3, β = 0.2, M= 4

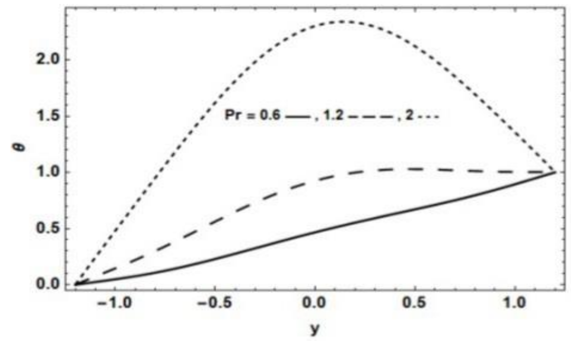

Fig.17. Effect of P_r on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β= 0.2, M= 4

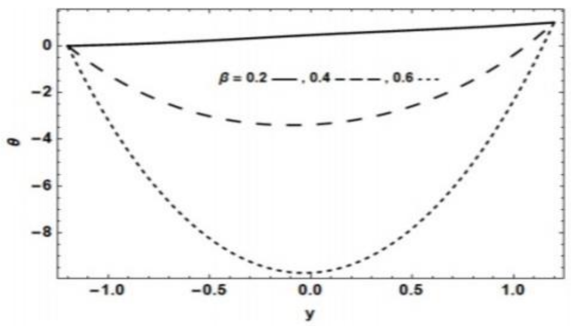

Fig.18 Effect of β on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, M=4

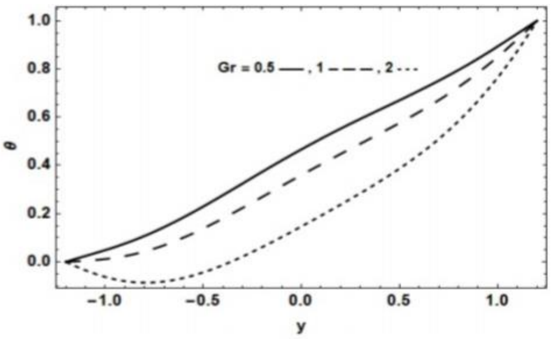

Fig.19. Effect of E_c on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

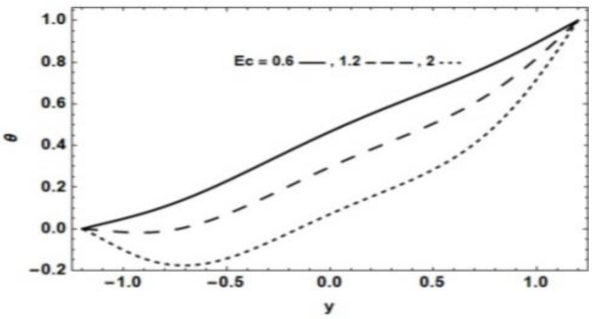

Fig.20. Effect of G_r on temperature θ
Where k= 0.6,F= 0.2, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

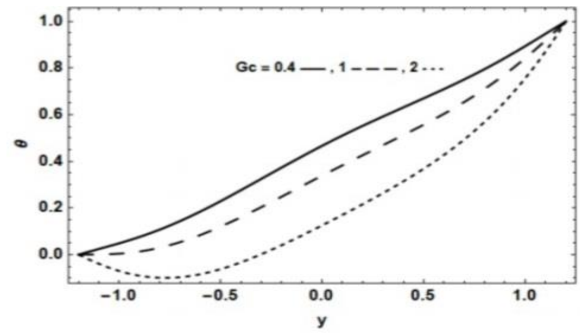

Fig.21. Effect of G_c on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

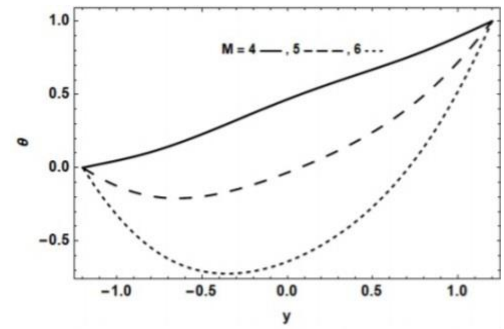

Fig.22. Effect of M on temperature θ
Where k=0.6,F=0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2

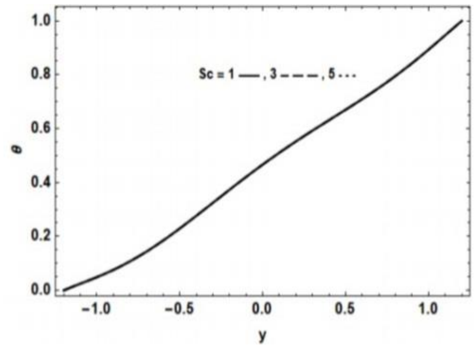

Fig.23. Effect of S_c on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c=0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_r= 3, β = 0.2, M= 4

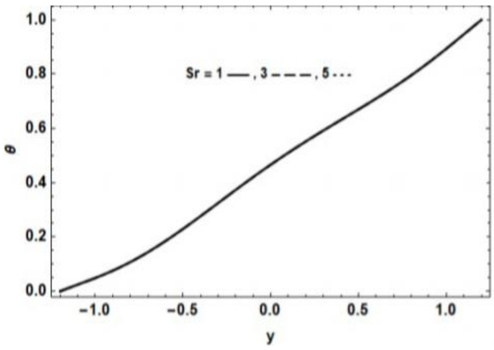

Fig.24. Effect of S_r on temperature θ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f=3, S_c= 3, β = 0.2, M= 4

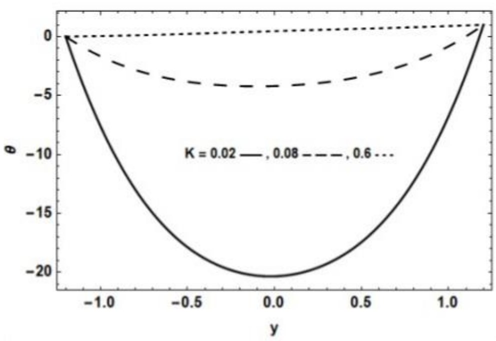

Fig.25. Effect of k on temperature θ
Where k=0.6, G_r =0.5, G_c=0.4, P_r=0.6,G=0.3, E_c=0.6 , λ_0=0.1, D_f=3, S_c=3, S_r=3, β = 0.2, M=4

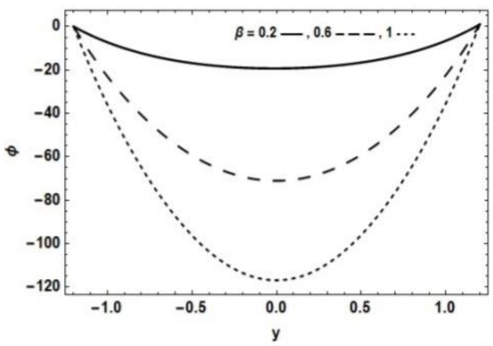

Fig.26. Effect of β on concentration φ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r=3, M= 4

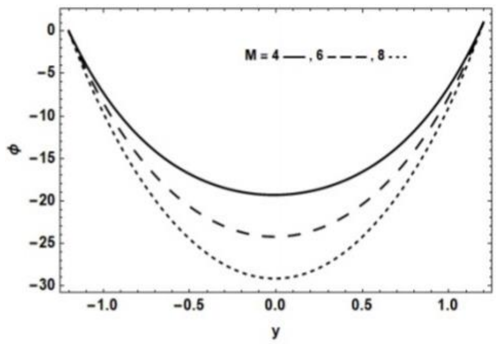

Fig.27. Effect of M on concentration φ
Where k=0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c=3, S_r=3, β = 0.2

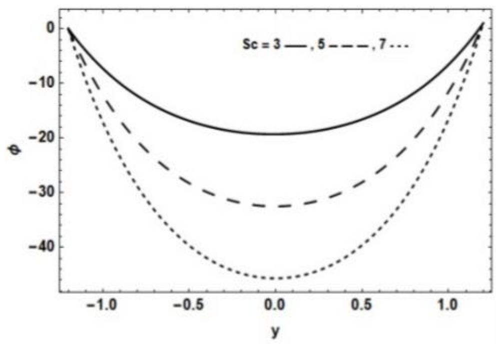

Fig.28. Effect of S_c on concentration φ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_r= 3, β = 0.2, M= 4

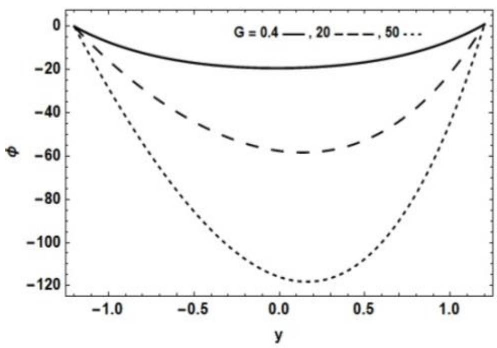

Fig.29. t. Effect of G on concentration φ Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

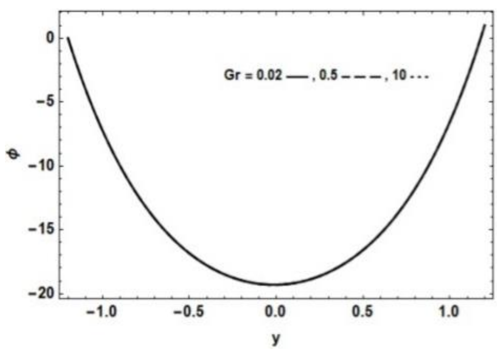

Fig.30.Effect of G_r on concentration φ
Where k= 0.6,F= 0.2,G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

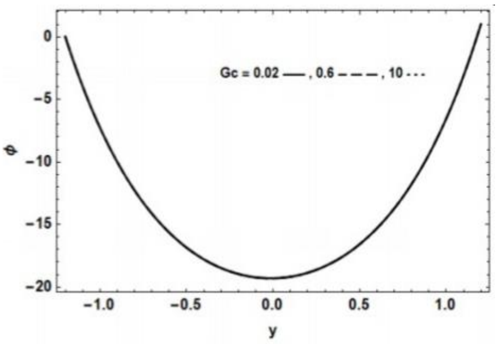

Fig.31. Effect of G_c on concentration φ
Where k= 0.6,F= 0.2, G_r= 0.5, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

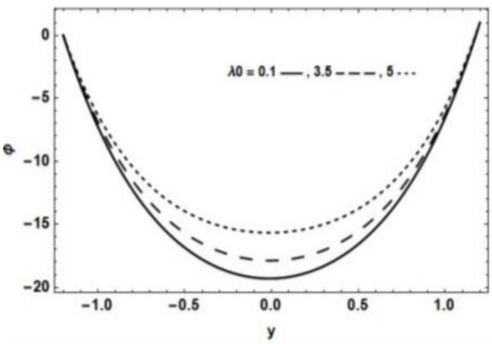

Fig.32. Effect of $\lambda_0$on concentration φ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

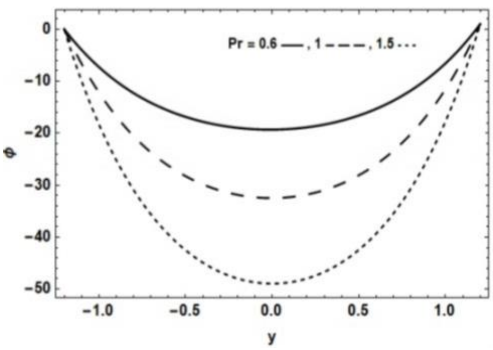

Fig.33. Effect of P_r on concentration φ
Where k= 0.6,F= 0.2, G_r= 0.5, G_c= 0.4, P_r= 0.6,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M= 4

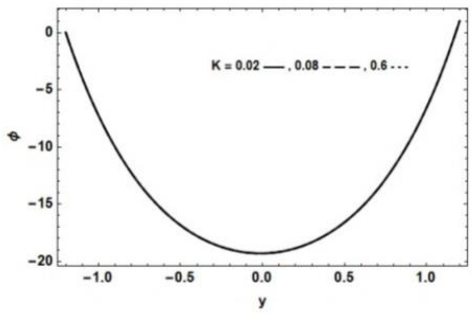

Fig.34. Effect of $k$on concentration φ
Where F= 0.2, G_r = 0.5, G_c= 0.4, P_r= 0.6
,G= 0.3, E_c= 0.6, λ_0= 0.1, D_f= 3, S_c= 3, S_r= 3, β = 0.2, M=4

5. Conclusion

This paper studied flow of the electrically conducting Casson fluid inside a vertical symmetric channel with flexible walls taking heat and mass transfer, Hall coefficient, viscous and elastic dissipation, heat generation and chemical reaction. The solution of non-linear partial differential equations which represent the velocity, temperature and concentration are obtained after applying the approximations of long wave length and low Reynolds number, with homotopy perturbation method. The influence of the physical parameters of the problem on these solutions are discussed numerically and graphically.

The very important results can be summarized as:

- The velocity u decreases by increasing magnetic field parameter M, non–Newtonian Casson parameter $\beta$and the chemical reaction parameterλ_0.
- The velocity u increases with non-darcian parameter, the Grashoff G_r , Prandtl P_r Eckert E_c numbers and heat generation G.
- The temperature θ increases with the non-darcian parameter F, heat generation G, and the Dufour parameter D_f .
- The temperature θ decreases with λ_0 and Prandtl number P_r .Increasing of Casson parameter, Eckert number E_c , Grashoff numberG_r , modified Grashoff number G_c and magntic parameter.
- The concentration φ decreases by increasing Casson parameter β, magnetic field parameter M, Soret $S_r$and Shimited numbers S_c and heat generations G
- The concentration φ increases with increasing the chemical reactions λ_0 .

6. Appendix:

$S_0 = \dfrac{A}{\left(1+(\frac{1}{\beta})\right)}$)$S_1 = (-1) - (\frac{S_0 h^2}{2}$

$S_2 = \left(1+(\frac{1}{\beta})\right)$ $S_3 = \left(M+(\frac{1}{k})\right)$

$S_4 = (\dfrac{F{S_0}^2}{4S_2})$ $S_5 = (\dfrac{S_0}{2S_2})(S_3 - FS_0h^2)$

$S_6 = \dfrac{(G_r+G_c)}{(2hS_2)}$ $S_7 = (S_4h^4) - \left(\dfrac{S_0S_3h^2}{2S_2}\right) - (S_6h)$

$S_8 = (\dfrac{S_4}{210})$ $S_9 = (\dfrac{S_5}{60})$

$S_{10} = (\dfrac{S_6}{24})$ $S_{11} = (\dfrac{S_7}{6})$

$S_{12} = (\frac{S_6 h^2}{12})$

$S_{14} = (\frac{ME_c P_r {S_0}^2}{120})$

$S_{16} = \frac{(10S_2 S_{14})}{M}$

$S_{18} = \frac{(GP_r)}{(12h)}$

$S_{20} = h^2[S_{14}h^4 - S_{17}h^2 + S_{19}]$

$S_{22} = 3hS_{21}$

$S_{24} = 3S_{21}h^3$

$S_{26} = \frac{(70S_8 S_9 F)}{S_2}$

$S_{28} = \frac{F(25S_9^2 + 42S_8 S_{11})}{S_2}$

$S_{30} = (7S_8 S_3) + (G_r S_{14})$

$S_{32} = \frac{(S_{30} + S_{31})}{S_2}$

$S_{34} = (5S_9 S_3) - (G_r S_{17})$

$S_{36} = \frac{(S_{34} + S_{35})}{S_2}$

$S_{30} = (7S_8 S_3) + (G_r S_{14})$

$S_{41} = (G_r S_{19}) - (G_c S_{22}) + (3S_3 S_{11})$

$S_{43} = \frac{(S_{41} + S_{42})}{S_2}$

$S_{45} = 4S_{12}S_{13}\mathrm{F}$

$S_{47} = \mathrm{F}S_{13}^2$

$S_{13} = \left(\frac{h^2}{60}\right)[(2S_4 h^4) + (5S_5 h^2) + (30S_7)]$

$S_{15} = 5S_{14}h^2$

$S_{17} = S_{15} - S_{16}$

$S_{19} = 15S_{14}h^4 + 3hS_{18}$

$S_{21} = \frac{\lambda_0}{(12h)}$

$S_{23} = S_{21}h^2$

$S_{25} = \frac{(49FS_8^2)}{S_2}$

$S_{27} = \frac{(56S_8 S_{10} F)}{S_2}$

$S_{29} = \frac{F(28S_8 S_{12} - 40S_9 S_{10})}{S_2}$

$S_{31} = \mathrm{F}[(16S_{10}^2) - (14S_8 S_{13}) + (30S_9 S_{11})]$

$S_{33} = \frac{F(20S_9 S_{12} - 24S_{10}S_{11})}{S_2}$

$S_{35} = \mathrm{F}[(9S_{11}^2) - (10S_9 S_{13}) - (16S_{10}S_{12})]$

$S_{37} = (G_r S_{18}) - (G_c S_{21}) - (4S_3 S_{10})$

$S_{40} = \frac{(S_{37} + S_{38})}{S_2}$

$S_{42} = \mathrm{F}[(4S_{12}^2) - (6S_{11}S_{13})]$

$S_{44} = (2S_3 S_{12}) - (G_r h^2 S_{18}) + (G_c S_{23})$

$S_{46} = \frac{(S_{44} - S_{45})}{S_2}$

$S_{48} = (S_3 S_{13}) + (G_r S_{20}) - (G_c S_{24})$

$S_{49} = \frac{(S_{47} - S_{48})}{S_2}$

$S_{51} = [\frac{(S_{25}h^{14})}{182} + \frac{(S_{26}h^{12})}{132} + \frac{(S_{28}h^{10})}{90} + \frac{(S_{32}h^8)}{56} + \frac{(S_{36}h^6)}{30} + \frac{(S_{43}h^4)}{12} + \frac{(S_{49}h^2)}{2}]$

$S_{53} = \frac{(S_{26})}{1716}$

$S_{55} = \frac{(S_{28})}{990}$

$S_{57} = \frac{(S_{32})}{504}$

$S_{59} = \frac{(S_{36})}{210}$

$S_{61} = \frac{(S_{43})}{60}$

$S_{63} = \frac{(S_{49})}{6}$

$S_{65} = (S_8 + S_{59})$

$S_{67} = (S_{62} - S_{10})$

$S_{69} = (S_{12} + S_{64})$

$S_{71} = ME_c P_r S_0$

$S_{73} = S_{71}(7S_8 - 5S_9)$

$S_{75} = (S_{73} + S_{74})$

$S_{77} = S_{71}(5S_9 - 3S_{11})$

$S_{79} = (S_{77} - S_{78})$

$S_{81} = \left[\frac{(24S_2 S_{71}S_{10})}{M}\right] + [GP_r S_{18}]$

$S_{50} = [\frac{(S_{27}h^{10})}{110} - \frac{(S_{29}h^8)}{72} - \frac{(S_{33}h^6)}{42} - \frac{(S_{40}h^4)}{20} - \frac{(S_{46}h^2)}{6}]$

$S_{52} = \frac{(S_{25})}{2730}$

$S_{54} = \frac{(S_{27})}{1320}$

$S_{56} = \frac{(S_{29})}{720}$

$S_{58} = \frac{(S_{33})}{336}$

$S_{60} = \frac{(S_{40})}{120}$

$S_{62} = \frac{(S_{46})}{24}$

$S_{64} = \frac{(S_{50})}{2}$

$S_{66} = (S_9 + S_{61})$

$S_{68} = [(\frac{S_0}{6}) + S_{11} + S_{63}]$

$S_{70} = (S_1 - S_{13} - S_{51})$

$S_{72} = 7S_8 S_{71}$

$S_{74} = \left[\frac{(-84S_2 S_{71}S_8)}{M}\right] + [GP_r S_{14}]$

$S_{76} = 4S_{10}S_{71}$

$S_{78} = \left[\frac{(40S_2 S_{71}S_9)}{M}\right] + [GP_r S_{17}]$

$S_{80} = S_{71}(2S_{12} + 4S_{10})$

$S_{82} = (S_{81} - S_{80})$

$S_{83} = S_{71}(S_{13} + 3S_{11})$

$S_{84} = \left[\frac{(-125S_2S_{71}S_{11})}{M}\right] + [GP_r S_{19}]$

$S_{85} = (S_{83} + S_{84})$

$S_{86} = (6P_r D_f S_{21} + GP_r S_{18}h^2)$

$S_{87} = S_{71}[(2S_{12}) - \left(\frac{4 S_2S_{12}}{M}\right)]$

$S_{88} = (S_{87} - S_{86})$

$S_{89} = \left(2P_r D_f S_{22} + S_{71}S_{13} + GP_r S_{20}\right)$

$S_{90} = h^2[\left(\frac{S_{76}h^4}{42}\right) + \left(\frac{S_{82}h^2}{20}\right) + \left(\frac{S_{88}}{6}\right)]$

$S_{91} = h^8[\left(\frac{S_{72}h^2}{90}\right) - \left(\frac{S_{75}}{56}\right)]$

$S_{92} = \frac{h^2}{2}[\left(\frac{S_{79}h^4}{15}\right) + \left(\frac{S_{85}h^2}{6}\right) + (S_{89})]$

$S_{93} = (S_{91} - S_{92})$

$S_{94} = \frac{S_{72}}{90}$

$S_{95} = \frac{S_{75}}{56}$

$S_{96} = \frac{S_{76}}{42}$

$S_{97} = \frac{S_{79}}{30}$

$S_{98} = \frac{S_{82}}{20}$

$S_{99} = \frac{S_{85}}{12}$

$S_{100} = \frac{S_{88}}{6}$

$S_{101} = \frac{S_{89}}{2}$

$S_{102} = (S_{97} - S_{14})$

$S_{103} = (S_{17} + S_{99})$

$S_{104} = (S_{100} - S_{18})$

$S_{105} = (S_{19} + S_{101})$

$S_{106} = \frac{1}{(2h)} + (S_{18}h^2) - (S_{90})$

$S_{107} = \frac{1}{2} + S_{20} + S_{93}$

$S_{108} = (S_cS_r)$

$S_{109} = (30S_{14}S_{108})$

$S_{110} = (\lambda_0 S_{21})$

$S_{111} = (\lambda_0 S_{22}) - (12S_{17}S_{108})$

$S_{112} = (6S_{18}S_{108}) - (\lambda_0 S_{23})$

$S_{113} = (2S_{19}S_{108}) - (\lambda_0 S_{24})$

$S_{114} = \frac{S_{109}}{30}$

$S_{115} = \frac{S_{110}}{20}$

$S_{116} = \frac{S_{111}}{12}$

$S_{117} = \frac{S_{112}}{6}$

$S_{118} = \frac{S_{113}}{2}$

$S_{119} = h^2[(S_{115}h^2) + (S_{117})]$

$S_{120} = h^2\begin{bmatrix}(S_{114}h^4)\\ +(S_{116}h^2) + (S_{118})\end{bmatrix}$

$S_{121} = (S_{21} + S_{117})$

$S_{122} = (S_{22} + S_{118})$

$S_{123} = \frac{1}{(2h)} - (S_{23}) - (S_{119})$

$S_{124} = \frac{1}{2} - (S_{24}) - (S_{120})$

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